THE POYNTING EFFECT.

THEORY, EXPERIMENT, PRACTICE.

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Abstract

Nonlinear behavior of an elastic circular cylinder undergoing twisting and stretching is investigated theoretically and experimentally. The influence of the Poynting effect, that is, the change in the length of the cylinder subjected to torque, on metrological properties of the rod-type force transducer with tensoresistors is considered.

1. Introduction

The Poynting effect is the change of the length of an elastic cylinder under the action of twisting. Common experience in wringing dry a roll of a wet cloth by twisting it appears to contain a clue to the possible mechanism of the effect. The generator of the cylinder becomes a helical line at an angle α and the length of the cylinder decreases by Δl = (cosα – 1) l. Assuming in what follows that α <<1, we have $\Delta l = -\alpha^2 l/2 = -\varphi^2 r^2/(2l)$, where φ is the angle of twisting of the cylinder and r is its radius.

In Poynting [1, 2] it was shown that

$$\Delta l = s\frac{\varphi^2 r^2}{2l}, \tag{1}$$

where s is an empirical coefficient ($s \approx 1$ for steel and $s \approx 1.5$ for brass).

Of great interest is not the formula itself that can be easily derived but rather the fact that $s > 0$. This means that a cylinder grows longer while being twisted. Even Lorentz ([3], 1927) could not avoid the influence of the wet cloth experience. He was *a priori* certain that $s < 0$, i.e., that the cylinder must become shorter but not longer. The imprints of the same influence are seen even in the Poynting formula (1): it is the coefficient 1/2 along with the s.

By the end of the 19th century, the torsion pendulum had already for about two hundred years been one of the most informative devices in experimental physics, and naturally non-linear effects of the twisting of elastic bodies could not escape the attention of researchers. Coulomb ([4], 1784) noted that the period of oscillations of such pendulum shortened under a large stretching force. Wertheim ([5], 1857), was the first who conducted appropriate experiments (examining the change of the volume of a pipe under twisting moment) and argued for the application of a nonlinear theory of twisting. 24 years later Bauschinger ([6], 1881) confirmed the main ideas of Wertheim. Kelvin ([7], 1865) and Tomlinson ([8], 1883) pointed to the change in the length of wires being twisted. Lastly, Poynting ([1], 1909) directly measured the change of length during twisting of stretched wires. 50 years later, theoretical explanations of the effect were found by Rivlin ([9], 1951), and Trusdell and Noll ([10], 1965).

60 years after the paper of Poynting [1], Allen and Saxl ([11], [12], 1969) investigated the twisting of metallic wires under considerable stretching loads. To a large extent they confirmed the work of Poynting and supplemented it with dynamic testing. In 1998 Dell'Isola, Ruta and Betra [13] analyzed the nonlinear behavior of a

prestretched prismatic bar subjected to twisting. They considered the bending of the bar to be a manifestation of the generalized Poynting effect.

The present study differs from the previous ones in two points:

1) The Poynting parameter *s* is determined theoretically, it is derived through the parameters of the constitutive equation of an elastic cylinder, and the results are confirmed experimentally.
2) The Poynting effect is considered as a factor that affects the accuracy of force transducers of the rod type; thus not only the torsion pendulum is an object of metrological refinement in terms of the nonlinear theory of twisting.

The body of this paper contains three main sections: **Theory, Experiment** and **Practice**.

The main topic in the **Theory** section is an effective scheme for the description of thermodynamics of a thin circular cylinder and its mathematical model. Both the description and the model are given in terms of the force, displacement and temperature. Twisting of the cylinder with its volume, length and radius being constant, generates forces the removal of which changes these variables. Three elastic moduli, namely, the shear modulus, Poisson's ratio and the Bridgman constant [14] are all that need be known about the material of the cylinder. A thick-walled (and a solid) cylinder is considered as a compact packet of thin-walled co-axial cylinders. The removal of forces that ensure the constancy of the volume, length and radius of each thin-walled cylinder causes the packet to become separated whereby the continuity of the cylinder's material is disturbed. To simplify the procedure of the removal of forces without loss of continuity of the material, we replace radial forces by mass forces. The section Theory proposes analytical expressions for the Poynting parameter *s* for thin-walled, thick-walled and solid cylinders as well as formulae for two components of the strain tensor of the lateral surface of the solid cylinder. In what follows these components will be measured with tensoresistors.

In the section **Experiment** the main idea of the model of the cylinder's material, namely the assertion that Poisson's ratio does not depend on the ball component of the strain tensor, is verified. Uniaxially stretching an elastic cylinder we determine the shear modulus, Poisson's ratio and the Bridgman constant. The value of the Bridgman constant is close to that was found by Bridgman for samples under hydrostatic pressure. The data obtained allow us to calculate the Poynting parameter and to estimate the Poynting effect as a factor influencing the accuracy of the rod-type force transducer itself. This estimation has been corroborated by experiment.

In the **Practice** section we estimate the percentage of the parts of the Poynting effect due to physical properties of the material and to kinematics of the deformation. We analyze the results of the papers [11] and [12] from the point of view of the theory presented. We propose a technique to offset the Poynting effect as a factor affecting the accuracy of readings of the force transducer of rod type.

In the **Conclusion**, we state that the effectiveness of the final result relates to the chosen form of the constitutive equations and to the specific algorithm of the experimentation. The discrepancy between our values of the parameter s and those given by Poynting [1] as well as by Allen and Saxl [11, 12] is due to the anisotropy of wires which were the object of their attention.

2. Notation

U The inner energy of the cylinder

S The entropy of the cylinder

Q The heat of the cylinder

T The temperature of the cylinder

V The volume of the cylinder

l The length of the cylinder

r The radius of the cylinder

α The angle of the inclination of the cylinder's generator

φ The angle of twisting of the cylinder

h The wall thickness of the cylinder

P The pressure

R The radially directed force

L The axial force

M The twisting torque

$\chi_1,\ldots,\chi_4$ The functions of the variables T, V, r, φ of the cylinder's thermodynamic state

c The heat capacity at constant V, r, l, φ

G The shear modulus at constant T, V, r, l

K The bulk modulus at constant T, r, l, φ

ν Poisson's ratio

a The Bridgman constant

s The Poynting parameter

B $B=4aG(1+\nu)/(3(1-2\nu))-1$

F The area of the cross-section of the cylinder

I The moment of inertia of the cross section of the cylinder

σ_z The normal stress on the ends of the cylinder

Δl The change of the length of the cylinder under torque

q	The equivalent mass force
ε_{11}, ε_{22}	The principal components of the strain tensor on the lateral surface of the solid cylinder
γ	The sensitivity factor of the measuring tensoresistors
Z	The resistance of the measuring tensoresistors
z_1, z_2	The relative change of resistances of the measuring tensoresistors
N	The output function of the digital-analogous processor that represents the force L
b	The amplifying factor
E	The voltage supplied to the bridge of the measuring tensoresistors
n	The factor of the analogous-digital transformer
k_1, k_2	The parameters of the second order approximation of L as a function of N
β	The coefficient of thermosensitivity
ΔL	The error of measurements of L as a result of the Poynting effect
m	The mass of the load
I_m	The moment of inertia of the load
H	The period of oscillations of the torsion pendulum
ΔH	The change in the period of oscillations due to additional mass

3. Theory

3.1. Consider a thin-walled circular cylinder. The thermodynamic state of the cylinder is determined by its volume V, its average radius r, the length l, the angle of twisting φ and the temperature T. The cylinder is loaded with the pressure P, the radial forces R, the axial forces L, and the torque M. The forces R are equally distributed over the middle surface of the cylinder of radius r and the forces L act on the plane bases of the cylinder.

Variations of the work δW and the heat δQ are expressed through variations of the state variables of the cylinder as follows:

$$\delta W = P\delta V + R\delta r + L\delta l + M\delta\varphi, \qquad \delta Q = \chi_1\delta V + \ + \chi_2\delta r + \chi_3\delta l + \chi_4\delta\varphi + c\delta T,$$

where $\chi_1,\ldots\chi_4$, and c are the functions of the state variables.

The first and the second laws of thermodynamics for the system under consideration reduce to the postulation of the two state functions: the internal energy U and the entropy S in the form

$$dU = \delta W + \delta Q, \qquad dS = \delta Q/T.$$

It follows that

$$dU = (P + \chi_1)\delta V + (R + \chi_2)\delta r + (L + \chi_3)\delta l + (M + \chi_4)\delta\varphi + c\delta T,$$

$$dS = \frac{\chi_1\delta V + \chi_2\delta r + \chi_3\delta l + \chi_4\delta\varphi + c\delta T}{T}.$$

The equalities for mixed derivatives of U and S in the state variables generate 20 differential equations. Only 6 of them, containing partial derivatives with respect to φ, relate to the Poynting effect (here T = const):

$$\frac{\partial}{\partial\varphi}(P + \chi_1) = \frac{\partial}{\partial V}(M + \chi_4), \qquad \frac{\partial(\chi_1/T)}{\partial\varphi} = \frac{\partial(\chi_4/T)}{\partial V},$$

$$\frac{\partial}{\partial\varphi}(R + \chi_2) = \frac{\partial}{\partial r}(M + \chi_4), \qquad \frac{\partial(\chi_2/T)}{\partial\varphi} = \frac{\partial(\chi_4/T)}{\partial r},$$

$$\frac{\partial}{\partial\varphi}(L + \chi_3) = \frac{\partial}{\partial l}(M + \chi_4), \qquad \frac{\partial(\chi_3/T)}{\partial\varphi} = \frac{\partial(\chi_4/T)}{\partial l}.$$

Analyzing the differential equations pair by pair we get the following three:

$$\frac{\partial P}{\partial \varphi} = \frac{\partial M}{\partial V}, \qquad \frac{\partial R}{\partial \varphi} = \frac{\partial M}{\partial r}, \qquad \frac{\partial L}{\partial \varphi} = \frac{\partial M}{\partial l}. \tag{2}$$

Now we state the constitutive relations for the cylinder in the form

$$\nu = const, \qquad G = \frac{Ml}{\varphi I}, \qquad \frac{V - V_0}{V_0} = \frac{P}{K_0}(1 + aP), \tag{3}$$

here ν is Poisson's ratio, G is the shear modulus, I is the moment of inertia of the cross-section of the cylinder, V_0 is the volume of the cylinder in the reference state, K_0 is the bulk modulus in the reference state, and a is the Bridgman constant [14].

From (3) it follows that $K = \frac{\partial P}{\partial V} V_0 = \frac{2(1+\nu)}{3(1-2\nu)} G$, where K is the bulk modulus. The essence of consitutive equations for the cylinder is this: the moduli G and K depend in a similar manner on the volume only so that ν=const. Note that the dependence of φ on M is linear only when V, r and l are constant. The nonlinear parts of the φ are small, they amount to a mere 0.1% of the nominal value of φ but it is they that determine the Poynting effect.

The shear modulus G and the geometrical parameters of the cylinder $V = 2\pi rlh$, $I = 2\pi r^3 h$, $F = 2\pi rh$ change when the cylinder is twisted. The character of change of h and G is found by the differentiation of V and G with respect to l, r, and V:

$$\frac{\partial G}{\partial V} = -\frac{4aG^2(1+\nu)}{3V(1-2\nu)}, \quad \frac{\partial h}{\partial l} = -\frac{h}{l}, \quad \frac{\partial h}{\partial r} = -\frac{h}{r}, \quad \frac{\partial h}{\partial V} = \frac{1}{2\pi rl}.$$

Here we neglect the difference between the reference and current values of the state parameters since taking of the derivatives is now over and anyway the difference is small. For the same reason we disregard the difference between the reference and current values of the moduli.

Now we get

$$\frac{\partial P}{\partial \varphi} = -\frac{\varphi r^2 G}{l^2} B, \qquad \frac{\partial R}{\partial \varphi} = \frac{\varphi r^2 G}{l^2} 4\pi l h, \qquad \frac{\partial L}{\partial \varphi} = -\frac{\varphi r^2 G}{l^2} 4\pi r h,$$

where $B=4aG(1+\nu)/(3(1\text{-}2\nu))-1$.

We assume that the reference state of the cylinder is natural. Integrating these equations with *V*, *r* and *l* being constant we obtain

$$P = -\frac{\varphi^2 r^2 G}{2l^2} B, \quad R = \frac{\varphi^2 r^2 G}{2l^2} 4\pi l h, \quad L = -\frac{\varphi^2 r^2 G}{2l^2} 4\pi r h.$$

Thus, when *V*, *r* and *l* are constant, twisting a thin-walled cylinder brings, the cylinder to the state as if it were compressed with the pressure *P*, stretched by the radial forces *R* and compressed with the axial forces *L*. The removal of the pressure and forces leads to a change in the values of *V*, *r*, and *l*. In particular, the length of the cylinder, *l*, increases by

$$\Delta l = \frac{\varphi^2 r^2}{2l} \frac{1}{2(1+\nu)} \left[B(1-2\nu) + 2(1+\nu) \right]$$

Comparing this with (1), we find that for a thin-walled cylinder

$$s = \frac{1}{2(1+\nu)} \left[B(1-2\nu) + 2(1+\nu) \right] \qquad (4)$$

3.2. Consider a thick-walled or solid cylinder as a compact packet of thin-walled co-axial cylinders. Twisting the packet through an angle φ at constant *V*, *r*, and *l* produces the above-mentioned pressure and forces in each thin-walled cylinder. The removal of the forces causes the packet to separate into distinct cylinders. It is hard to "collect" the packet without violating the condition of continuity of the material. It makes sense to replace the radial forces by the equivalent mass forces:

$$q = R/(2\pi r l h) - dP/dr = \varphi^2 r G (B+1)/l^2 .$$

The removal of the surface and mass forces (the Lame problem) brings about elongation of the cylinder and an increase of its volume. On the plane sides of the cylinder there remains the stress σ_z that does not vanish at $L = 0$. For a solid cylinder we get $\sigma_z = \frac{\varphi(2\tilde{r}^2 - r^2)}{8l^2} G[B(1-2\nu) + 2(1+\nu)]$, where $\tilde{r}$ is the current radius. The removal of this stress causes some deplanation of the cross-sections of the cylinder. Neglecting the deplanation, we obtain by analogy with (4) for a thick-walled cylinder:

$$s = \frac{1 + (r_1/r_2)^2}{4(1+\nu)} \left[B(1-2\nu) + 2(1+\nu) \right],$$

where r_1 and r_2 are the inner and outer radii of the cylinder.

For a solid cylinder we have

$$s = \frac{1}{4(1+\nu)} \left[B(1-2\nu) + 2(1+\nu) \right].$$

In what follows we shall need expressions for the principal components of the strain tensor in the middle part of the solid cylinder's lateral surface:

$$\varepsilon_{11} = \frac{\varphi^2 r^2}{8l^2} \frac{1}{(1+\nu)} \left[B(1-2\nu) + 2(1+\nu) \right], \tag{5}$$

$$\varepsilon_{22} = \frac{\varphi^2 r^2}{8l^2} \left[(B-1)(1-2\nu) - \frac{\nu}{1+\nu}(B+2) \right]. \tag{6}$$

4. Experiment

We experiment with a force transducer whose thermomechanics was studied earlier [15]. The elastic element of the transducer is a 90-mm long solid cylinder of radius 10 mm made of a steel alloy. In the middle part of the cylinder's surface a measuring tensorosette with four grids and a temperature gauge of the resistance type are placed. The components of the measuring rosette are assembled as a symmetrical bridge. In the rosette, the first and third tensoresistors represent the strain tensor component ε_{11}, while the second and fourth ones represent the component ε_{22}. A machine of direct loading of 100кN and a computer system for taking readings that provides 5 significant digits (100000 readings) of the signal of the force transducer are used. A device for twisting of 200 N·m is employed.

An essential feature of the method of measuring is that nearly all the information is taken from the object of investigation itself, that is, from the rod-type force transducer. The coefficient of tensosensitivity γ is the only parameter obtained from an external data source. We have used experimental techniques of tensometry that provide greater accuracy [16,17].

In the initial stage of the experiment the elastic element of the transducer is in the state of one-axial stress. The two principal components of the strain tensor, ε_{11} and ε_{22}, are components of the input function for the main channel for measuring force L. The input transformer of the channel is the tensorosette. At the output of the channel there is an analogous-digital transformer supplying the function N to the computer bus; $N = (bEn/4)(z_1 - z_2)(1 - z_1 - z_2)$, where b is the gain factor of the channel, E is the voltage on the bridge of the tensorosette, n is the transformation coefficient of the ADT and z_1 and z_2 are relative changes in the resistance of the rosette that represent the strain components ε_{11} and ε_{22}. According to [16], $z_1 = \gamma\varepsilon_{11}\left[1 + (\gamma - 1)\varepsilon_{11}/2\right]$, $z_2 = -\nu\gamma\varepsilon_{11}\left[1 - \nu(\gamma - 1)\varepsilon_{11}/2\right]$. These lead to the expression $N = (bEn/2)\gamma(1+\nu)\varepsilon_{11}\left[1 - (1-\nu)(\gamma + 1)\varepsilon_{11}/2\right]$. The dependence of the L being measured on N is approximated by $L = k_1 N(1 + k_2 N)$. Thence, when $\varepsilon_{11} \to 0$, it follows that

$$G = \frac{k_1 bEn\gamma}{4F},$$

where F is the area of the cross-section of the elastic element.

The corrections for the temperature change are made by a computer program: $N := N\left[1+\beta\left(T-T_0\right)\right]$, where T is the current temperature of the elastic element, T_0 is its initial temperature, and β is the coefficient of thermo-sensitivity.

To determine Poisson's ratio ν, the tensorosette is broken at any of the four points of mating of the tensoresistors. Two equal resistors supplement the half-bridge, obtained in this manner. They shunt the first and third tensoresistors that represent in the rosette the strain component ε_{11}. The output function N, up to a factor, is [17] $N=\varepsilon_{11}\{[Y/(Z+Y)]^2[1+((\gamma-1)/2-\gamma Z/(Z+Y))\varepsilon_{11}]-\nu[1-\nu(\gamma-1)\varepsilon_{11}/2]\}$, where Z is the resistance of the measuring grid, and Y is the shunt resistance. When $\nu = const$ there is a Y that can be determined by the equation $\nu = \left[Y/(Y+Z)\right]^2$ for which $N \sim \varepsilon_{11}^2 \sim L^2$. For Z=400.10 Ω and Y=449.53 Ω this dependence is observed in the test ($L \le 10^5$ N) within the accuracy of five significant digits of the value of L. This means that $\nu = const$ with an accuracy to the 4th significant digit.

To determine the Bridgman constant, the measured force L is represented through the ball component of the stress tensor, that is to say, through the pressure P: $L = 3PF\left(1-\nu\varepsilon_{11}\right)^2$. The volume component ε of the strain tensor that corresponds to the pressure P is $\varepsilon = \left(V-V_0\right)/V_0 = \left(1+\varepsilon_{11}\right)\left(1-\nu\varepsilon_{11}\right)^2 - 1$. It then follows that

$$\varepsilon = \frac{3(1-2\nu)P}{2G(1+\nu)}\left[1+\frac{3P}{2G(1+\nu)}\left(\frac{(1-\nu)(\gamma+1)}{2}-\frac{\nu(2-\nu)}{1-2\nu}-2\nu-\frac{2Fk_2G(1+\nu)}{k_1}\right)\right]$$

Comparing this with the third equation from (3) we arrive at

$$a=\frac{3}{2}\frac{1}{G(1+\nu)}\left[(1-\nu)\frac{\gamma+1}{2}-\frac{\nu(2-\nu)}{1-2\nu}-2\nu-2F\frac{k_2G}{k_1}(1+\nu)\right]$$

In the second stage of the experiment, the elastic element of the transducer is twisted by the torque *M*. The results obtained allow us to estimate the torque *M* as a factor affecting the accuracy of measurements of force *L*. Up to the small quantities of the second order, the systematic measurement error $\Delta L = 2FG(\varepsilon_{11}-\varepsilon_{22})$, where the strains are given by (5) and (6). Thus we have

$$\Delta L=\frac{\pi M^2}{F^2G}\left[2\nu^2(B-1)+3(1+\nu)\right]. \qquad (7)$$

The goal of the experimentation with the force transducer is to obtain estimates for the Poynting coefficient *s* and the systematic error in the measurements of ΔL. But first we determined the following parameters:

$\gamma = 2.100 \pm 0.004$, $F=(3.140\pm0.003)\cdot10^{-4}\,m^2$, $b=100.0\pm0.1$, $E=5.000\pm0.001$ V, $N=26214\pm5\ 1/V$, $k_1=3.6586\pm0.0002$ N, $k_2=(-1.86\pm0.004)\cdot10^{-7}$, $\beta=(-3.50\pm$ $\pm\,0.04)\cdot10^{-4}\ 1/{}^0K$, $G=(0.802\pm0.002)\cdot10^{11}\ Pa$, $\nu=0.280\pm0.0005$, $a=(0.40\pm$ $\pm\,0.02)\cdot10^{-10}\ 1/Pa$, $B=11.5\pm0.6$.

For the transducer under investigation we have got $s=1.48\pm0.06$. The systematic error ΔL in the measurements of *L* determined theoretically and experimentally is given in the Table.

Table

Influence of the Poynting effect on the accuracy of measurement of force *L*

by a force transducer of the rod type

	Experiment	Theory

M, $N\,m$	ΔL, N	ΔL, N
98	22 ± 1	21 ± 1
196	90 ± 2	84 ± 4

5. Practice

5.1. In practice, one often neglects the nonlinear behavior of the force transducer assuming $L=k_1N$. Had we done so we would have obtained the following results: $a= - 0.08\cdot 10^{-10}$ 1/Pa, that is, in absolute value, 5 times smaller than the real value of the Bridgman constant, and s=0.30, 5 times smaller than the magnitude found in our experiments and 3 times smaller than the value found by Poynting [1]. Under the same conditions, the value of ΔL decreases 1.6 times in comparison with the above theoretical value and 1.7 times compared with our experiment.

5.2. Let us estimate the effect (in %) of the physical and the kinematical parts of nonlinearity on the values of the main parameters.

Assume that the Bridgman constant that represents the physical part of nonlinearity is equal to zero, a=0, then $k_2= 0.31\cdot 10^{-7}$. This gives 16% of the $k_2 = 1.86\cdot 10^{-7}$ that was obtained during graduated tests of the force transducer with a direct loading device. In other words, under uni-axial stretching, the kinematical part of nonlinearity amounts to 16% of the full value and the physical one comes out at 84%.

A similar estimate for s, when the elastic element of the transducer is twisted, yields the following results: 34% for the kinematics and 66% for the physics of the process. In the estimation of ΔL, kinematics gives 68% and physics 32%.

5.3. If we assume that the Bridgman constant for structural steels and alloys is the same, then it will be possible to interpret the results of Allen and Saxl [11,12] from the point of view of the above theory. The assumption is approximately valid since, according to Bridgman [14], a = (0.37 ± 0.07)10^{-10} Pa^{-1} for *Al* and a = (0.36± 0.07) · 10^{-10} Pa^{-1} for *Fe*. We remind the reader that for steel a = (0.40 ± 0.02) · 10^{-10} Pa^{-1}.

Allen and Saxl [11,12] investigated the effect of the stretching load on the value of the period of oscillations H of a torsion pendulum. The stretching load is provided by added weights of total mass m. The weights increase the moment of inertia I_m of the pendulum and the length l of the pendulum suspension wire. At the same time the radius r of the wire and its shear modulus G are diminished. This leads to an increase in the period of tortional oscillations of the pendulum. To determine the relationship of ΔH with ΔI_m, ΔG, Δl and Δr, Allen and Saxl [11] used a formula of Poynting [1] that represents nonlinear behavior of the torsional pendulum. Poynting derived the formula from a linear ordinary differential equation that follows from the Lagrange equation of motion. Actually, the real form of the corollary of Lagrange's equation is the nonlinear differential equation

$$I_m\ddot{\varphi} + m\left(sr^2/l\right)^2\left(\ddot{\varphi}\varphi^2 + 2\dot{\varphi}^2\varphi\right) + \left(\pi G r^4/(2l)\right)\left(1 - 2mgs/\pi G r^2\right)\varphi = 0,$$

where s is the Poynting parameter and g is the gravitational acceleration.

For the simplification done by Poynting there is a reason: indeed, for a low-frequency pendulum such as that of Allen and Saxl, the second addend is by its norm 8 to 10 decimal orders times smaller than the first and the third addends. From the simplified equation it follows that $H = 2\pi\left(2I_m l/\left(\pi G r^4[(1-2mgs/(\pi G r^2)]\right)\right)^{1/2}$. For minor disturbances (of the order of 10^{-4}) introduced by mass m

$\Delta H/H = 0.5\left(\Delta I_m/I_m + \Delta l/l - 4\Delta r/r - \Delta G/G + 2mgs/\left(\pi G r^2\right)\right)$. With the results of Section 3 taken into account this formula becomes

$$\Delta H/H = 0.5\Delta I_m/l + \left(gm/\left(\pi r^2\right)\right)\left[(1+4\nu)/\left(4G(1+2\nu)\right) + a/3 + s/G\right].$$

Combining the value of $a = 0.40\cdot 10^{-10}$ Pa^{-1} that we found and the data from Allen and Saxl [11], $\nu = 0.35$, $G = 0.675\cdot 10^{11}$ Pa, $r = 6.35\cdot 10^{-4}$ m, and $s = 0.943$, and neglecting the first addend we get $\Delta H/H = 2.52\cdot 10^{-4}\cdot m$ whereas Allen and Saxl [11] obtained $\Delta H/H = 2.31\cdot 10^{-4}\cdot m$. A detailed analysis of the results of [12] has shown that the latter ratio needs to be changed to $\Delta H/H = 2.45\cdot 10^{-4}\cdot m$, which practically coincides with our result.

5.4. In the case of high accuracy of the rod-type transducer (say, within 0.1%), this limit of error can in practice be compromised by the Poynting effect. Consider weighing a mass of 10000 kg. Let the moment of inertia I_m of the weight be with respect to the vertical axis 2500 kg·m^2. The weight rotates about the axis with the period of rotational oscillations $H \approx 10c$ and the amplitude angle $\varphi = 0.1$. The torque moment that is applied to the rod element of the transducer is $\mathrm{M} = 4\pi^2 \mathrm{I_m} \varphi \mathrm{H}^2 \approx 100\mathrm{N\cdot m}$, which corresponds to $\Delta L \approx 20N$, that is, 0.2% of the value measured. To avoid the adverse influence of the Poynting effect on the accuracy of measurements of the force it is necessary to equip the elastic element with another tensorosette for measuring the torque moment. It is necessary to use the output of this thermorosette in accordance with the formula (7) to correct the readings with the aid of a computer program or in some other way.

6. Conclusion.

The effectiveness of the final result is due to the chosen form of the constitutive equations and to the specific algorithm of the experimentation. An attempt to use the constitutive law by Murnaghan [18] and the pertinent data of other experiments given in [19] yielded unsatisfactory results. We cannot expect success even if we improved the Murnaghan equations by introducing the fourth and fifth degrees of the invariants of the strain tensor disregarding the limits of metrological capacity of measuring devices.

The fact that the three-parameter model of the material and modest mathematical tools brought the theory and the experiment to a good agreement is not a chance event. The mathematical model is physically substantiated and the mathematical tools are adequate to the problem in hand.

The reason for the difference between our values of the Poynting parameter *s* and the results of other authors lies probably in the anisotropy of stretched wires which were observed by them [1,11].

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